Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

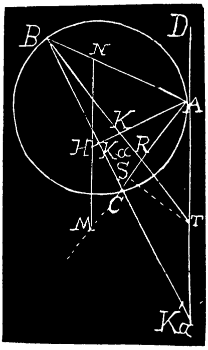
Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

median meets the side BC. Take $AN=AC$, and $AM=AB$. Then in $\triangle ANM$, AH, the median, is the symmedian of $\triangle ABC$. Lines BC, and MN are antiparallel. Also, since $\measuredangle ABC=\measuredangle BAD$, each being measured by ½arc AC, the lines BC and AK_a' are antiparallel. Wherefore MN is parallel to the tangent line AK_a'.



Now we have a pencil of four rays AB, AH, AC, AK_a' in which one ray AH bisects a line parallel to its conjugate, and included between the other pair of conjugate rays; hence the pencil is harmonic, and any line, as BK_a', drawn across the pencil will cut out an harmonic range $\{BC,\ K_aK_a'\}$. Q. E. D.

(3). Draw ST perpendicular to AC at its middle point S; draw BT, and it is a symmedian line (Halsted: Syn. Geom. §648.), hence it passes through Grebe's point (or Lemoine's Point) K. Now as $A\{BC,\ K_aK_a'\}$ is an harmonic pencil, $\{BR,\ KT\}$ is an harmonic range; whence $S\{BR,\ KT\}$ is an harmonic pencil. Draw altitude BP, and it is || to ray ST, and is therefore bisected by the ray SK, the conjugate of ST. Therefore the line joining the middle point, S, of a side, and the middle point of the altitude to that side passes through Grebe's point.

Q. E. D.

59. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Show that the tangent plane at any point of the surface $a^2x^2+b^2y^2+c^2z^2=2bcyz+2acxz+2abxy$ intersects the surface $ayz+bzx+cxy=0$ in two straight lines at right angles to one another.

Solution by the PROPOSER.

The tangent plane at $(x',\ y',\ z')$ to $F=0$..........................(1) is

$$(x-x')\frac{dF}{dx'}+(y-y')\frac{dF}{dy'}+(z-z')\frac{dF}{dz'}=0 \quad \text{..........................(2).}$$

Here $F=a^2x^2+b^2y^2+c^2z^2-2bcyz-2acxz-2abxy$..................(3).

$$\frac{dF}{dx'}=2a(ax'-by'-cz'),\ \frac{dF}{dy'}=2b(-ax'+by'-cz'),$$

$$\frac{dF}{dz'}=2c(-ax'-by'+cz') \quad \text{.....................(4).}$$

Then (2) becomes by aid of (3),

$$a(ax'-by'-cz')x+b(-ax'+by'-cz')y+c(-ax'-by'+cz')z=0 \quad \text{.............(5).}$$

It may be shown that the condition that

$lx+my+nz=0$ ....(6) cuts $ayz+bxz+cxy=0$........... .....(7)

in two straight lines including a right angle is $amn+bnl+clm=0$...........(8).

Comparing (5) and (6), $l=a(ax'-by'-cz')$, $m=b(-ax'+by'-cz')$, $n=c(-ax'-by'+cz')$, and (8) becomes

$$abc\{a^2x'^2-(by'-cz')^2+b^2y'^2-(cz'-ax')^2+c^2z'^2-(ax'-by')^2\}=0 \quad\text{..........(9)},$$

an identity by aid of (3).

Also solved by *HENRY HEATON* and *J. SCHEFFER*.

PROBLEMS.

65. Proposed by I. J. SCHWATT, Ph. D., Professor of Mathematics, University of Pennsylvania, Philadelphia, Pennsylvania.

Prove in a pure geometrical way the following:

The axes of the ellipse isogonal to Lemoine's line with respect to a triangle (Steiner's ellipse) are parallel to Simson's lines belonging to the extremities of Brocard's Diameter.

66. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

The locus of points whose polars with respect to a given parabola touch the circle of curvature at the vertex is an equilateral hyperbola.

MECHANICS.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

32. Proposed by OTTO CLAYTON, A. B., Fowler, Indiana.

The wheel of a wind pump has 60 fans, each turned at an angle of 45° to the direction of the axis, and each having 150 square inches exposed to the wind. If the wind blows with a velocity of V and the wheel rotates with velocity ω, what is the component of force or pressure along the axis if it is turned at an angte α to the direction of the wind assuming the resistance of the wheel in turning to be R?

Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas-Texas.

Let A=projecting area of fans exposed to the wind, in square feet,
V=velocity of wind in feet per second,
H=horse hower of pump,
R=extreme radius of fans in feet,